

## FRAZIONI ALGEBRICHE - 1

Determinare l'insieme di definizione delle seguenti frazioni algebriche a valori in  $\mathbb{Q}$ .

1.  $\frac{5+a}{a}$ ;  $\frac{2x+5}{x-2}$ ;  $\frac{3a+2}{2a}$   $[\mathbb{Q}^*; \mathbb{Q} - \{2\}; \mathbb{Q}^*]$
2.  $\frac{x}{x(x+1)}$ ;  $\frac{7b+3}{-b+1}$ ;  $\frac{x(x+9)}{x-3}$   $[\mathbb{Q} - \{-1, 0\}; \mathbb{Q} - \{1\}; \mathbb{Q} - \{3\}]$
3.  $\frac{10(a+1)}{(a+2)^2}$ ;  $\frac{x}{x^2+1}$ ;  $\frac{x(x+1)(x+2)}{x^2(x+3)}$   $[\mathbb{Q} - \{-2\}; \mathbb{Q}; \mathbb{Q}^* - \{-3\}]$
4.  $\frac{a+1}{a(a+2)}$ ;  $\frac{5x+9}{7x-3}$ ;  $\frac{a(a+1)^2}{1-a^2}$   $[\mathbb{Q}^* - \{-2\}; \mathbb{Q} - \{\frac{3}{7}\}; \mathbb{Q} - \{-1, +1\}]$
5.  $\frac{(x-1)^3}{(x+1)^4}$ ;  $\frac{ab(ab-3)}{ab+1}$ ;  $\frac{xy+3}{x^2y^2+5}$   $[\mathbb{Q} - \{-1\}; a, b \in \mathbb{Q}, ab \neq -1; \mathbb{Q}]$
6.  $\frac{a+b}{a^2-b^2}$ ;  $\frac{x}{x^7-1}$ ;  $\frac{(x-3)(x+5)}{(16x-2)^2}$   $[\forall a, b \in \mathbb{Q} : a \neq \pm b; \mathbb{Q} - \{1\}; \mathbb{Q} - \{\frac{1}{8}\}]$
7.  $\frac{x^3(x+2)^2}{(11x-7)^3}$ ;  $\frac{8x}{64x^2}$ ;  $\frac{x^3(x+1)^4}{1-x^3}$   $[\mathbb{Q} - \{\frac{7}{11}\}; \mathbb{Q}^*; \mathbb{Q} - \{1\}]$
8.  $\frac{x^4-16}{x^2-9}$ ;  $\frac{-11x-5}{11x}$ ;  $\frac{3}{x^4+1}$   $[\mathbb{Q} - \{-3, +3\}; \mathbb{Q}^*; \mathbb{Q}]$
9.  $\frac{5x^2(x+1)^2(x-3)}{8+x^2+x^4}$ ;  $\frac{x^2+5x+3}{x^2-7x+12}$   $[\mathbb{Q}; \mathbb{Q} - \{3, 4\}]$
10.  $\frac{b^3+8}{a(b-a)}$ ;  $\frac{x-12}{(x^2+y^2)^2}$   $[b \neq a, a \neq 0; x \neq 0 \text{ e } y \neq 0]$
11.  $\frac{x^2-x}{x^2-1}$ ;  $\frac{5x^2+x-6}{x(x^2+x-6)}$   $[x \neq \pm 1; x \neq 0, x \neq -3, x \neq 2]$
12.  $\frac{(2a+2)(a-1)}{(2a-2)(a+1)}$ ;  $\frac{5x^2+5}{x^4-1}$   $[a \neq \pm 1; x \neq \pm 1]$
13.  $\frac{2-\frac{1}{x-1}}{\frac{x^2+x}{3x-3}} \cdot \frac{x}{x-3}$ ;  $\frac{(x^2+2x+1)\left(\frac{1}{2}x+\frac{3}{2}\right)}{(x+1)(x^2-9)}$   $[x \neq 0, x \neq \pm 1, x \neq 3; x \neq -1, x \neq \pm 3]$

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Semplificare le seguenti frazioni algebriche <sup>(1)</sup>.

- |                                    |  |  |  |   |
|------------------------------------|--|--|--|---|
| 15. $\frac{ab^2}{2b};$             | 3ab;<br>3a   | ab <sup>3</sup> c <sup>3</sup> ;<br>ac <sup>2</sup>                  | 16a <sup>2</sup> ;<br>4ab              | È $\left[ \frac{ab}{2}; b; b^3c; 4\frac{a}{b} \right]$                            |
| 16. $\frac{2a^2b^2c}{abc^2};$      | 4abc;<br>16a   | 4a;<br>16a <sup>2</sup> b  | abc;<br>3a <sup>2</sup> c <sup>2</sup> | $\left[ 2\frac{ab}{c}; \frac{bc}{4}; \frac{1}{4ab}; \frac{b}{3ac} \right]$        |
| 17. $\frac{12x^4y^2z}{30x^2y^2w};$ | -12xyz <sup>2</sup> ;<br>4x <sup>2</sup> y <sup>2</sup> z  | 30x <sup>2</sup> y;<br>25y <sup>3</sup>                              |  | $\left[ \frac{2x^2z}{5w}; -\frac{3z}{xy}; \frac{6x^2}{5y^2} \right]$              |
| 18. $\frac{2a-4b}{6a};$            | 12a <sup>2</sup> -3ab;<br>6ab  | 3xy;<br>2x <sup>2</sup> -xy  |  | $\left[ \frac{a-2b}{3a}; \frac{4a-b}{2b}; \frac{3y}{2x-y} \right]$                |
| 19. $\frac{ab}{a^2b^2-ab};$        | 2a <sup>2</sup> ;<br>4a <sup>4</sup> -2a <sup>2</sup>  | 3xy;<br>3x <sup>2</sup> yz-3xy                                       |  | $\left[ \frac{1}{ab-1}; \frac{1}{2a^2-1}; \frac{1}{xz-1} \right]$                 |
| 20. $\frac{x^2+2x}{x};$            | 3a <sup>2</sup> -6ab;<br>a-2b  | 2x <sup>2</sup> -3xy;<br>4x <sup>2</sup>                             |  | $\left[ x+2; 3a; \frac{2x-3y}{4x} \right]$  |
| 21. $\frac{x+y}{x^2+xy};$          | a-2b;<br>2a <sup>2</sup> -4ab  | xy-y <sup>2</sup> ;<br>ay-ax   |  | $\left[ \frac{1}{x}; \frac{1}{2a}; -\frac{y}{a} \right]$                          |
| 22. $\frac{a-b}{b-a};$             | x-2y;<br>2y-x  | -a-b;<br>a+b   |  | [-1, -1; -1]  |
| 23. $\frac{7x-14}{x^2-4};$         | 6x <sup>2</sup> +12xy;<br>x <sup>2</sup> +4y <sup>2</sup> +4xy                                     | 15a <sup>3</sup> b <sup>2</sup> ;<br>12a <sup>2</sup> b <sup>3</sup> |  | $\left[ \dots; \frac{6x}{x+2y}; \dots \right]$                                    |
| 24. $\frac{2a^2-2a}{a^2-1};$       | 3a-3ax;<br>3a <sup>2</sup> -3ax <sup>2</sup>   | 6x+6y;<br>3x <sup>2</sup> -3y <sup>2</sup>                           |  | $\left[ \frac{2a}{a+1}; \frac{1-x}{a-x^2}; \frac{2}{x-y} \right]$                 |
| 25. $\frac{3a^2-12}{a^2-4a+4};$    | ax <sup>2</sup> -by <sup>2</sup> ;<br>a <sup>2</sup> x <sup>4</sup> -b <sup>2</sup> y <sup>4</sup> | a <sup>4</sup> -1;<br>a <sup>6</sup> -1                              |  | $\left[ \frac{3(a+2)}{a-2}; \frac{1}{ax^2+by^2}; \frac{a^2+1}{a^4+a^2+1} \right]$ |
| 26. $\frac{a^2-a}{5-5a};$          | 6x-6;<br>7x-7  | 8a <sup>4</sup> b+24a <sup>3</sup> b;<br>5a <sup>2</sup> b+15ab      |  | $\left[ -\frac{a}{5}; \frac{6}{7}; \frac{8}{5}a^2 \right]$                        |

<sup>(1)</sup> Conviene specificare, per esercizio, l'insieme di esistenza delle singole frazioni.

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27.  $\frac{x^2 - 4y^2}{x^2 - 4xy + 4y^2}$ ;  $\frac{x^2 - 16}{x^2 - 8x + 16}$ ;  $\frac{x^2 - 9y^2}{x^2 + 6xy + 9y^2}$ ;  $\left[ \frac{x+2y}{x-2y}, \frac{x+4}{x-4}, \frac{x-3y}{x+3y} \right]$
28.  $\frac{ax+ay}{ax-ay}$ ;  $\frac{ax+ay}{3x+3y}$ ;  $\frac{x^3+2x^2}{x^2+4x+4}$ ;  $\left[ \frac{x+y}{x-y}, \frac{a}{3}, \frac{x^2}{x+2} \right]$
29.  $\frac{a^4-b^4}{a^2+b^2}$ ;  $\frac{a^2-8a+16}{a^2-3a-4}$ ;  $\frac{6a^2-29ab+9b^2}{4a^2-36ab+81b^2}$ ;  $\left[ a^2-b^2; \frac{a-4}{a+1}; \frac{3a-b}{2a-9b} \right]$
30.  $\frac{a^3-a^2-2a}{a^3-4a}$ ;  $\frac{a^3-6a^2+5a}{a^3-a}$ ;  $\frac{2x^2-18y^2}{3x^2+18xy+27y^2}$ ;  $\left[ \frac{a+1}{a+2}; \frac{a-5}{a+1}; \frac{2(x-3y)}{3(x+3y)} \right]$
31.  $\frac{5x^4-5y^4}{3x^2+3y^2}$ ;  $\frac{42a^3-30a^2b}{35ab^2-25b^3}$ ;  $\frac{14a^2-7ab}{10ac-5bc}$ ;  $\left[ \frac{5}{3}(x^2-y^2); \frac{6a^2}{5b^2}; \frac{7a}{5c} \right]$
32.  $\frac{a^3-b^3}{4a-4b}$ ;  $\frac{x^3+8y^3}{x+2y}$ ;  $\frac{b^2-16}{b^2-8b+16} \cdot \left[ \frac{1}{4}(a^2+ab+b^2); x^2-2xy+4y^2; \frac{b+4}{b-4} \right]$
33.  $\frac{x^2-9y^2}{x^2+6xy+9y^2}$ ;  $\frac{x^2+y^2-z^2+2xy}{x^2-y^2+z^2+2xz}$ ;  $\frac{a^3-b^3}{a^4-b^4}$ ;  $\left[ \frac{x-3y}{x+3y}; \frac{x+y-z}{x-y+z}; \frac{a^2+ab+b^2}{(a+b)(a^2+b^2)} \right]$
34.  $\frac{p^4-q^4}{p^5-p^3q^2}$ ;  $\frac{ax+bx+ay+by}{a^2+ab}$ ;  $\frac{x^3-25xy^2}{2x^2-12xy+10y^2}$ ;  $\left[ \frac{p^2+q^2}{p^3}; \frac{x+y}{a}; \frac{x(x+5y)}{2(x-y)} \right]$
35.  $\frac{x^3+4x^2+4x}{x^3+6x^2+12x+8}$ ;  $\frac{a^2-2a}{a^2-3a+2}$ ;  $\left[ \frac{x}{x+2}; \frac{a}{a-1} \right]$
36.  $\frac{x^2-(a-b)x-ab}{x^3+bx^2+ax+ab}$ ;  $\frac{x^2+(a+b)x+ab}{x^2+(a+c)x+ac}$ ;  $\left[ \frac{x-a}{x^2+a}; \frac{x+b}{x+c} \right]$
37.  $\frac{x^2-(a+b)x+ab}{x^2-(a-c)x-ac}$ ;  $\frac{(x+a)^2-(b+c)^2}{(x+b)^2-(a+c)^2}$ ;  $\left[ \frac{x-b}{x+c}; \frac{x+a-b-c}{x+b-a-c} \right]$
38.  $\frac{x^4+(2b^2-a^2)x^2+b^4}{x^4+2ax^3+a^2x^2-b^4}$ ;  $\frac{x^2+(a+b+c)x+(a+b)c}{a^2+2ab+b^2-x^2}$ ;  $\left[ \frac{x^2-ax+b^2}{x^2+ax-b^2}; \frac{x+c}{a+b-x} \right]$
39.  $\frac{x^2+3x+2}{x^2+6x+5}$ ;  $\frac{x^2+10x+21}{x^2-2x-15}$ ;  $\frac{a^2-4a+4}{a^2-5a+6}$ ;  $\left[ \frac{x+2}{x+5}; \frac{x+7}{x-5}; \frac{a-2}{a-3} \right]$
40.  $\frac{a^2-3a+2}{a^2-1}$ ;  $\frac{x^2-7x+12}{x^2-8x+15}$ ;  $\frac{x^2-x-20}{x^2+x-30}$ ;  $\left[ \frac{a-2}{a+1}; \frac{x-4}{x-5}; \frac{x+4}{x+6} \right]$
41.  $\frac{a^5-a^4b+a^2b^3-ab^4}{a^4+a^3b-a^2b^2-ab^3}$ ;  $\frac{3a^2+a-2}{3a^3+a^2+a-2}$ ;  $\left[ \frac{a^2-ab+b^2}{a+b}; \frac{a+1}{a^2+a+1} \right]$
42.  $\frac{t^3-t^2-t+1}{t^4-t^3-3t^2+5t-2}$ ;  $\frac{y^2-(3b-c)y+2b(b-c)}{y^2-b^2-c^2+2bc}$ ;  $\left[ \frac{t+1}{(t-1)(t+2)}; \frac{y-2b}{y+b-c} \right]$
43.  $\frac{(x-a)^2-(y+a)^2}{x^2-y^2}$ ;  $\frac{ax-ay+bx-by+a+b}{ax-ay-bx+by+a-b}$ ;  $\left[ \frac{x-y-2a}{x-y}; \frac{a+b}{a-b} \right]$
44.  $\frac{ay-2y+3a-6}{ay-2y-3a+6}$ ;  $\frac{a^4-9a^3b+27a^2b^2-27ab^3}{a^4-6a^3b+9a^2b^2}$ ;  $\left[ \frac{y+3}{y-3}; \frac{a-3b}{a} \right]$
45.  $\frac{b^5-2b^4-16b+32}{b^4-2b^3-4b^2+8b}$ ;  $\frac{p^2-7p+12}{p^2-8p+15}$ ;  $\left[ \frac{b^2+4}{b}; \frac{p-4}{p-5} \right]$